PHYSICAL REVIEW E VOLUME 60, NUMBER 2 AUGUST 1999

Effect of a magnetic field on sonoluminescence

Kyuichi Yasui
National Industrial Research Institute of Nagoya, 1-1 Hirate-cho, Kita-ku, Nagoya 462-8510, Japan

(Received 10 September 1998; revised manuscript received 3 May 1999)

The effect of a magnetic field on single-bubble sonoluminescence in water reported experimentally by Young, Schmiedel, and Kang [Phys. Rev. Lett. **77**, 4816 (1996)] is studied theoretically. It is suggested that bubble dynamics is affected by the magnetic field because moving water molecules of the liquid suffer torque due to the Lorentz force acting on their electrical dipole moment, which results in the transformation of some of the kinetic energy into heat. It is shown that the magnetic field acts as if the ambient pressure of the liquid were increased. It is suggested that the effect increases as the amount of the liquid water increases. It is predicted that nonpolar liquid such as dodecane exhibits no effect of the magnetic field.
[S1063-651X(99)10808-0]

PACS number(s): 78.60.Mq

I. INTRODUCTION

In 1990, Gaitan [1] reported single-bubble sonoluminescence (SBSL) where a stably oscillating bubble driven by the standing acoustic wave in the liquid emits light at the collapse. The pulse width of the light is experimentally measured to be 40–350 ps [2,3]. The spectrum of the light can be fitted by the blackbody formula with the effective temperature of 6000–50 000 K [4,5]. In 1996, Young, Schmiedel, and Kang [6] reported experimental evidence of the effect of magnetic field on SBSL intensity. They reported that the SBSL intensity decreases as the magnetic flux density increases [6]. It is also reported that the acoustic pressure required for SBSL increases as the magnetic flux density increases; the acoustic pressure for SBSL ranges from 1.3–1.5 atm when magnetic field is off, while it ranges from 1.5–1.8 atm when the magnetic flux density is 10 T [6]. These results indicate a dramatic modification of bubble dynamics by magnetic fields [6]; however, it has not yet been understood theoretically.

II. THEORY

Physical properties of liquid water at rest is hardly affected by magnetic fields [7]. However, moving water molecules interact with the magnetic field by the Lorentz force acting on their electrical-dipole moment. A water molecule suffers a torque of $\mathbf{N}=\mathbf{P}_{\mathrm{H_2O}}\times(\mathbf{v}\times\mathbf{B})$ from the magnetic field $\mathbf{B}$ when it moves with the velocity $\mathbf{v}$, where $\mathbf{P}_{\mathrm{H_2O}}$ is the electrical-dipole moment of a water molecule ($|\mathbf{P}_{\mathrm{H_2O}}|=6.5\times10^{-30}\,\mathrm{C\,m}$). Thus a moving water molecule has the potential energy of $U=\int\mathbf{N}\cdot d\varphi=-\mathbf{P}_{\mathrm{H_2O}}\cdot(\mathbf{v}\times\mathbf{B})$, where $d\varphi$ is the element of the rotation vector, and tends to lie in the direction of $(\mathbf{v}\times\mathbf{B})$. It implies that a part of the kinetic energy of each water molecule of the liquid is transferred to the rotational energy of the molecule, which is finally transferred to heat by the frictional force acting between water molecules of the liquid. The energy transferred from the kinetic energy to the rotational energy per water molecule is crudely proportional to $U=-\mathbf{P}_{\mathrm{H_2O}}\cdot(\mathbf{v}\times\mathbf{B})$, which is finally transferred to heat. Thus the total energy transferred from the kinetic energy of liquid water to heat by the rotational motion due to the interaction with the magnetic field (ΔE_B) in time Δt is crudely estimated by

$$\Delta E_B=C\nu_l\Delta t\int_R^L|\mathbf{P}_{\mathrm{H_2O}}||\mathbf{v}||\mathbf{B}|\,\frac{\rho N_A}{M_{\mathrm{H_2O}}}\,4\pi r^2dr, \tag{1}$$

where C is a constant $(0<C\ll1)$, ν_l is the frequency of the thermal vibration of the liquid (the frequency of the ‘‘phonon’’ of the liquid), L is the distance between the bubble and the wall of the liquid container, R is the bubble radius, $\mathbf{v}$ is the macroscopic velocity of the liquid water, ρ is the liquid density, N_A is the Avogadro number, $M_{\mathrm{H_2O}}$ is the molar weight of water, and r is the radius from the center of the bubble. The term ν_l in Eq. (1) means that the direction of each water molecule is always perturbed by thermal vibrations of the liquid. The coefficient C is proportional to the probability of the displacement of the dipole (a water molecule) from the direction of $(\mathbf{v}\times\mathbf{B})$ by the perturbation per thermal vibration of the surrounding liquid; hence, it is much smaller than 1. The velocity of the liquid water is given by $|\mathbf{v}|=R^2\dot{R}/r^2$ [8], where the dot denotes the time derivative (d/dt). Thus

$$\Delta E_B=4\pi C\nu_l|\mathbf{P}_{\mathrm{H_2O}}||\mathbf{B}|\,\frac{\rho N_A}{M_{\mathrm{H_2O}}}\,R^2\dot{R}L\Delta t, \tag{2}$$

where $R\ll L$ is used.

Now we will derive the equation of bubble radius (R) including the above effect. First, we calculate the kinetic energy of the liquid (E_k) [8].

$$E_k=\int_R^L\frac{1}{2}\rho|\mathbf{v}|^2 4\pi r^2dr=2\pi\rho R^3\dot{R}^2, \tag{3}$$

where $R\ll L$ is used. Thus the change of the kinetic energy per unit time is

$$\frac{dE_k}{dt}=2\pi\rho R^2\dot{R}(3\dot{R}^2+2R\ddot{R}). \tag{4}$$

Next, the work done by the bubble (W) is calculated [8].

$$W=\int_{R_0}^{R}(p_B-p_\infty)4\pi r^2 dr, \tag{5}$$

where R_0 is the ambient bubble radius, p_B is the liquid pressure at the bubble wall, and p_∞ is the pressure at a point remote from the bubble [$p_\infty=p_0+p_s(t)$, where p_0 is the ambient pressure and p_s is a nonconstant component such as a sound field]. Thus the rate of the work done by the bubble is

$$\frac{dW}{dt}=(p_B-p_\infty)4\pi R^2\dot{R}. \tag{6}$$

The energy balance is given by

$$\frac{dW}{dt}=\frac{dE_k}{dt}+\frac{dE_B}{dt}, \tag{7}$$

which leads to the equation of bubble radius (R).

$$R\ddot{R}+\frac{3}{2}\dot{R}^2=\frac{1}{\rho}\left[p_B-\left(p_s(t)+p_0+C\nu_l|\mathbf{P}_{\mathrm{H_2O}}|\frac{\rho N_A}{M_{\mathrm{H_2O}}}|\mathbf{B}|L\right)\right]. \tag{8}$$

When the magnetic field is off ($\mathbf{B}=\mathbf{0}$), Eq. (8) reduces to the Rayleigh-Plesset equation [8]. From Eq. (8), it is seen that the magnetic field acts as if the undisturbed ambient pressure (p_0) were increased. It is also seen that the effect increases as the magnetic flux density increases. The effect also increases as the size of the liquid container increases. In other words, it increases as the amount of the liquid water increases. It is also concluded that if the molecules of the liquid have no electrical dipole moment the effect of magnetic field vanishes; for example, dodecane for which single-bubble sonoluminescence has been observed [5].

III. NUMERICAL CALCULATION

In Eq. (8), the numerical value of the coefficient C is unclear. In order to determine C, numerical simulations of a bubble collapse under the experimental condition [6] are performed based on the quasiadiabatic compression model [9]. The model used in the present simulation is fully described in Ref. [9]. The equation of bubble radius (R) used is the modified Keller equation, in which the effect of the liquid compressibility and that of evaporation and condensation of water vapor at the bubble wall are taken into account [10]. In the model [9], the pressure is assumed to be spatially uniform inside a bubble and the temperature is assumed to be spatially uniform except at the thermal boundary layer near the bubble wall whose thickness is $n\lambda'$ where $n=7$ [11] and λ' is the mean free path of gas molecules. In the model [9], the effect of nonequilibrium evaporation and condensation of water vapor at the bubble wall, that of thermal conduction both inside and outside a bubble, and that of chemical reactions inside a bubble, are taken into account.

In another paper by the author [12], it is clarified that the light-emission mechanism for a noble gas bubble is a radiative recombination of electrons and ions when the degree of ionization is fairly high and that the intensity of the light (I) is estimated by

TABLE I. Calculated results for various ambient pressures (p_0). The frequency and amplitude of the acoustic wave are 43.1 kHz and 1.68 atm, respectively [6]. The ambient bubble radius is assumed to be 3 μm. $R_{\max}$ is the maximum bubble radius, $R_{\min}$ is the minimum bubble radius, $T_{\max}$ is the maximum bubble temperature, $I_{\max}$ is the maximum intensity of the emitted light, and ''pulse width'' is that of the light.

p_0	1.0 atm	1.05 atm	1.1 atm
$R_{\max}$	46.7 μm	43.1 μm	39.6 μm
$R_{\min}$	0.47 μm	0.48 μm	0.48 μm
$T_{\max}$	25 300 K	23 100 K	20 600 K
$I_{\max}$	18 mW	7 mW	2 mW
pulse width	60 ps	60 ps	60 ps

$$I=\tfrac{4}{3}\pi R^3 q^2 N^2 \sigma_{fb}\overline{v_e}h\overline{\nu}, \tag{9}$$

where q is the degree of ionization, N is the number density of noble gas molecules, σ_{fb} is the cross section of radiative recombination, $\overline{v_e}$ is the mean velocity of electrons, h is the Planck constant, and $\overline{\nu}$ is the mean frequency of the emitted light [$h\overline{\nu}=(3/2)kT$ is assumed, where k is the Boltzmann constant and T is the bubble temperature]. The degree of ionization (q) is calculated by the Saha equation [12].

In Table I, the calculated results are listed for $p_0=1.0$, 1.05, and 1.1 atm. The frequency and amplitude of the acoustic wave are 43.1 kHz and 1.68 atm, respectively [6]. The ambient bubble radius (R_0) is assumed to be $R_0=3$ μm. It is seen that as the ambient pressure (p_0) increases the maximum bubble radius decreases and the collapse becomes milder as seen in Fig. 1. Accordingly, the maximum bubble temperature decreases and the maximum light intensity decreases. Young, Schmiedel, and Kang [6] reported that the number of photons per burst decreases from 3×10^7 to 0.6×10^7 as the magnetic flux density increases from zero to 6 T. Thus it is expected from Table I that the effect of the magnetic field of 6 T corresponds to that of the ambient pressure of 1.1 atm. Thus the coefficient C in Eq. (8) is crudely estimated to be $C\sim10^{-7}$ using the value $\nu_l\sim10^{12}$ Hz [13], which is consistent with the requirement $C\ll1$.

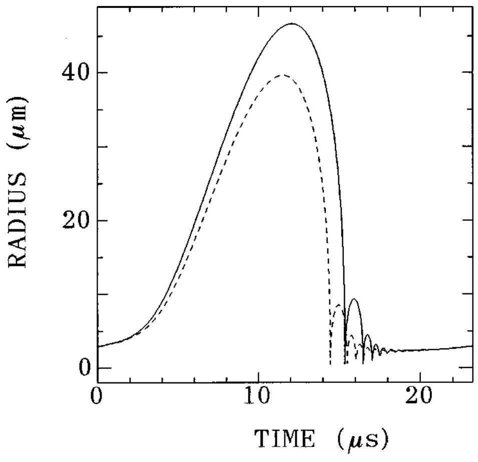

FIG. 1. The bubble radius (R) as a function of time for one acoustic cycle for $p_0=1.0$ atm (solid line) and $p_0=1.1$ atm (dotted line).

Recently, Kondic, Yuan, and Chan [14] studied theoretically the effect of ambient pressure on SBSL. They reported that the acoustic pressure required for SBSL increases as the ambient pressure increases [14] that is consistent with the present prediction that magnetic field acts as if the ambient pressure were increased because it is reported [6] that the acoustic pressure for SBSL increases as the magnetic flux density increases.

IV. CONCLUSION

The effect of a magnetic field on single-bubble sonoluminescence in water reported experimentally [6] is studied theoretically. It is shown that some of the kinetic energy of liquid water is transferred to heat by the interaction with the magnetic field because moving water molecules suffer torque from the magnetic field due to the Lorentz force acting on their electrical-dipole moment. It is concluded that the magnetic field acts as if the ambient pressure were increased. It is suggested that the effect of the magnetic field increases as the amount of the liquid water increases. Results of the numerical calculations based on a quasiadiabatic compression model of a bubble collapse [9] are consistent with the present analysis. It is predicted that nonpolar liquid such as dodecane exhibits no effect of magnetic field.

[1] D. F. Gaitan, Ph.D. thesis, University of Mississippi, 1990; D. F. Gaitan, L. A. Crum, C. C. Church, and R. A. Roy, J. Acoust. Soc. Am. **91**, 3166 (1992).

[2] B. Gompf, R. Günther, G. Nick, R. Pecha, and W. Eisenmenger, Phys. Rev. Lett. **79**, 1405 (1997).

[3] R. Hiller, S. J. Putterman, and K. R. Weninger, Phys. Rev. Lett. **80**, 1090 (1998).

[4] R. Hiller, Ph.D. thesis, University of California, 1995.

[5] B. P. Barber, R. A. Hiller, R. Löfstedt, S. J. Putterman, and K. R. Weninger, Phys. Rep. **281**, 65 (1997).

[6] J. B. Young, T. Schmiedel, and W. Kang, Phys. Rev. Lett. **77**, 4816 (1996).

[7] N. E. Dorsey, *Properties of Ordinary Water-Substance* (Reinhold, New York, 1940).

[8] T. G. Leighton, *The Acoustic Bubble* (Academic, London, 1994).

[9] K. Yasui, Phys. Rev. E **56**, 6750 (1997).

[10] K. Yasui, J. Phys. Soc. Jpn. **65**, 2830 (1996).

[11] K. Yasui, J. Acoust. Soc. Am. **98**, 2772 (1995).

[12] K. Yasui, Phys. Rev. E **60,** 1754 (1999).

[13] P. A. Egelstaff, *An Introduction to the Liquid State* (Academic, London, 1967).

[14] L. Kondic, C. Yuan, and C. K. Chan, Phys. Rev. E **57**, R32 (1998).